SHREE SAD VIDYA MANDAL INSTITUTE OF TECHNOLOGY
BHARUCH

Fluid Mechanics

- A overview

Ashish J. Modi
M.E. (Jet Propulsion & Gas Turbine)
ashishjmodi@gmail.com
+91 94271 01077

Statement of Laws for a System

- Newton's First Law of Motion:
  - ☐ *Every body preserves in its state of rest or of constant velocity unless it is compelled to change that state by some external action impressed thereon.*

$$\Sigma F = 0 \text{ and } \Sigma M = 0$$

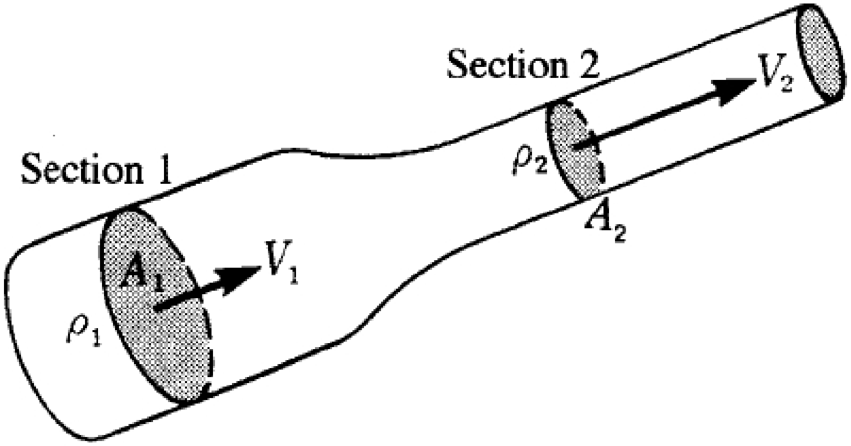
- Newton's Second Law of Motion:
  - ☐ *The rate of change of momentum of a body is proportional to the impressed action.*
  - ☐ The momentum implied may be linear or angular and the corresponding actions are force and moment respectively.
- Newton's Third Law of Motion:
  - ☐ *To every action, there is always an equal and opposite reaction.*

External force

- External forces acting on an element are classified as surface force and body force.
  - surface force: a force expressible as a stress multiplied by area of a surface e.g., pressure force.
  - body force: a force proportional to the volume of the fluid element e.g., gravity force.

Continuity Equation

- For the pipe shown in Fig. whose diameter decreases between sections 1 and 2, which have cross-sectional areas A_1 and A_2 respectively, and at which the mean velocities are V_1 and V_2 and the densities ρ_1 and ρ_2 respectively,



Mass flow rate passing through any section is constant

Continuity Equation

- In steady flow, the mass flow per unit time passing through each section does not change, even if the pipe diameter changes. This is the law of conservation of mass.

$$A_1 v_1 \rho_1 = A_2 v_2 \rho_2$$

$$A v \rho = \text{constant}$$

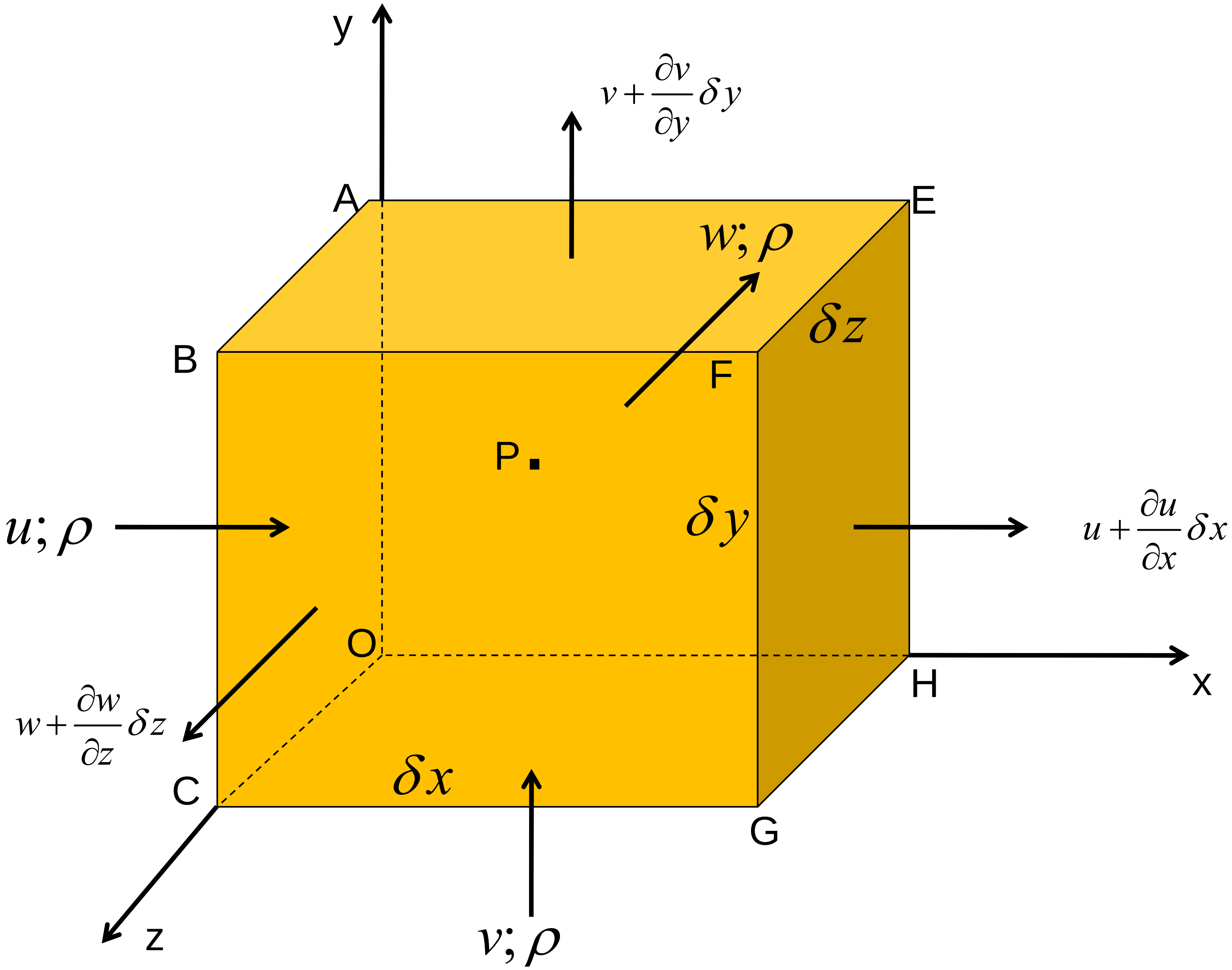
- If the fluid is incompressible, e.g. water, with ρ being effectively constant, then

$$A v = \text{constant}$$

Continuity Equation – Differential Form

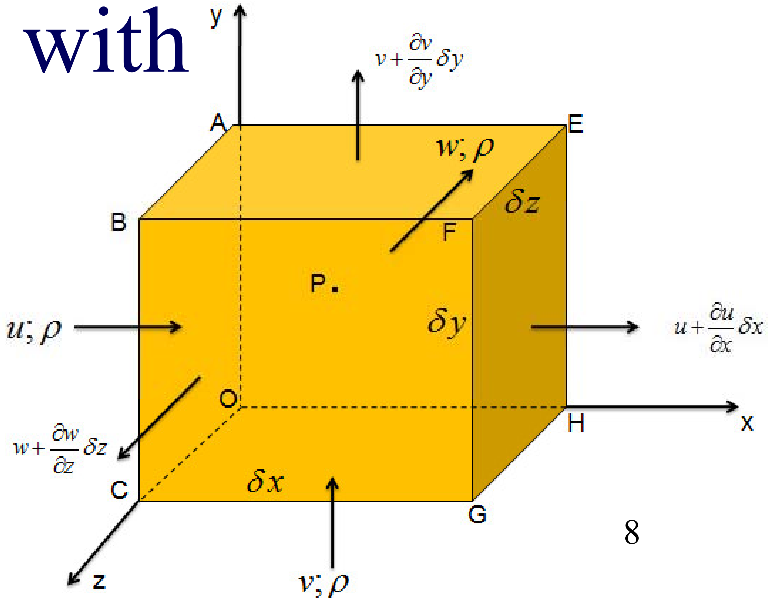
- The law of conservation of mass for a differential control volume requires that the net mass rate leaving the control volume plus the rate of accumulation of mass within the control volume must equal zero.

Continuity Equation – Differential Form



- Consider the point enclosed by an elementary CV and efflux of mass across each surface as well as rate of accumulation within CV is considered.
- Assuming that fluid enters ABCO with a velocity u and leaves EFGH with a velocity $u + \frac{\partial u}{\partial x} \bullet \delta x$
- If fluid is compressible than enters with ρ at ABCO and leaves with EFGH with

$$\rho + \frac{\partial \rho}{\partial x} \bullet \delta x$$



For an incompressible flow of a field,

mass rate entering face ABCO $= \rho u \cdot \delta y \bullet \delta z$

mass rate leaving face EFGH $= \rho\left(u + \frac{\partial u}{\partial x}\delta x\right)\delta y \cdot \delta z$

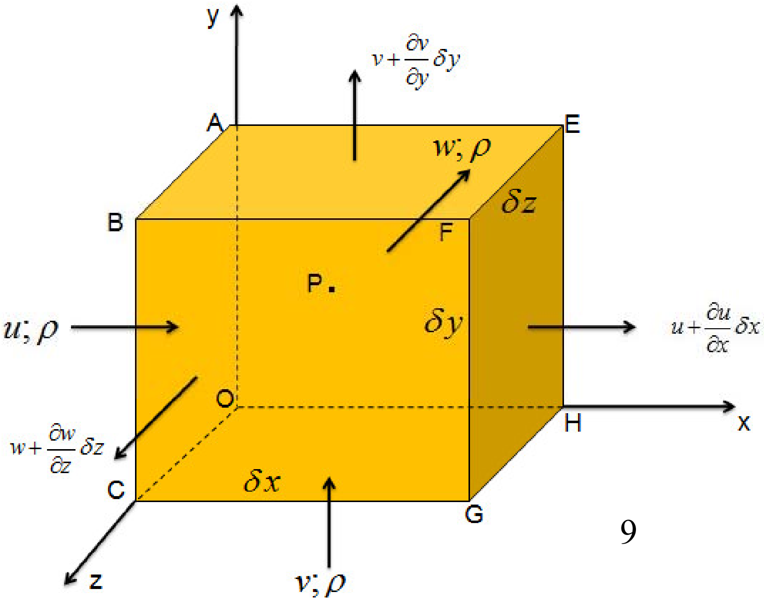
$$= \rho u \cdot \delta y \cdot \delta z + \rho \frac{\partial u}{\partial x}\delta x \cdot \delta y \cdot \delta z$$

Hence, the net mass rate leaving in the x-direction, by difference,

$$= \rho \frac{\partial u}{\partial x}\delta x \cdot \delta y \cdot \delta z = \rho \frac{\partial u}{\partial x}\delta V$$

Similarly in y and z direction,

$$\rho \frac{\partial v}{\partial y}\delta V \text{ and } \rho \frac{\partial w}{\partial z}\delta V$$



Rate of accumulation of mass within the C.V.,

$$\frac{\partial}{\partial t}(\rho\delta V)=\frac{\partial\rho}{\partial t}\delta V$$

Since ρ is constant for incompressible flow, this term vanishes showing that there can be no accumulation of mass with time within the control volume in an incompressible flow.

By mass balance, the net rate of mass efflux across the constant volume should be zero.

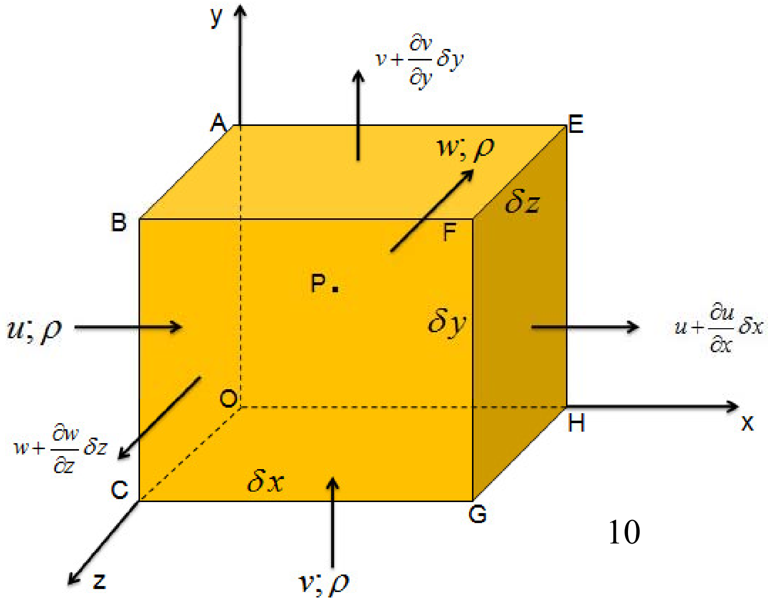
$$\rho\frac{\partial u}{\partial x}\delta V+\rho\frac{\partial v}{\partial y}\delta V+\rho\frac{\partial w}{\partial z}\delta V=0$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

equation is for 3 D incompressible flow.

The equation is valid for steady or unsteady flow because there is no $\partial/\partial t$ term in it.

Equation valid for both viscous and inviscid fluid.



The equation of continuity for incompressible flow may be rewritten as –

$$\nabla \cdot U = 0$$

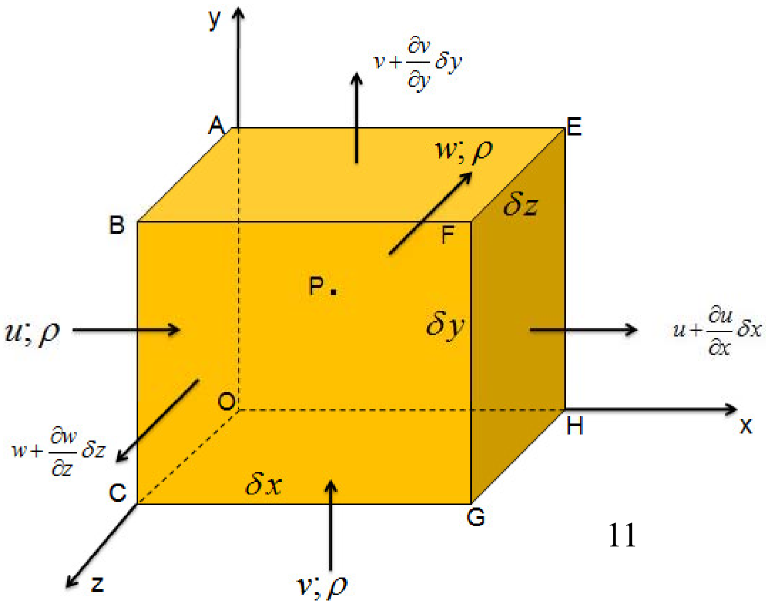
$$\nabla \equiv \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k$$

$$U = ui + vj + wk$$

For 2 D incompressible flow, e.g. plane flow in the x-y plane, the continuity equation reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

because $\frac{\partial}{\partial z} \equiv 0$



For a compressible flow of a field,

mass rate entering face ABCO $= \rho u \cdot \delta y \bullet \delta z$

mass rate leaving face EFGH

$$= \left(\rho + \frac{\partial \rho}{\partial x}\delta x\right)\left(u + \frac{\partial u}{\partial x}\delta x\right)\delta y \cdot \delta z$$

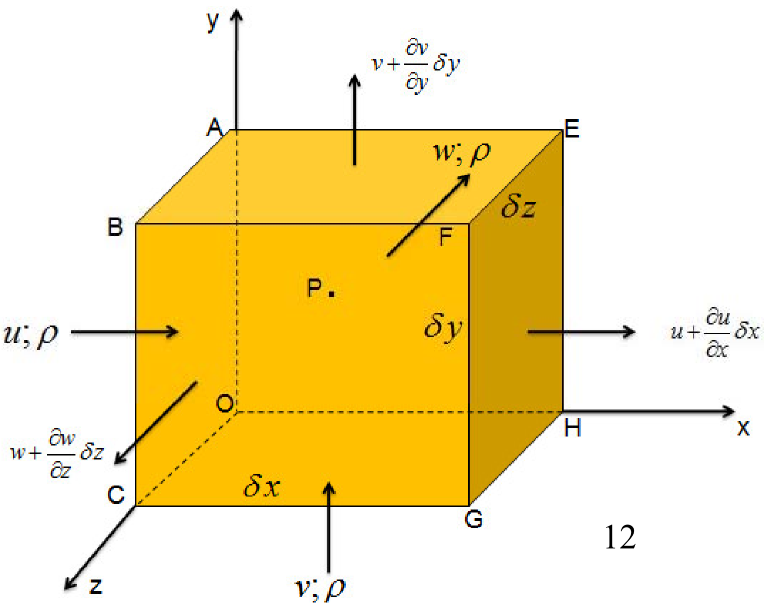
$$= \rho u \cdot \delta y \cdot \delta z + \rho \frac{\partial u}{\partial x}\delta x \cdot \delta y \cdot \delta z + u \frac{\partial \rho}{\partial x}\delta x \cdot \delta y \cdot \delta z + \text{ smaller terms}$$

Hence, the net mass rate leaving in the x-direction,

$$= \rho \frac{\partial u}{\partial x}\delta x \cdot \delta y \cdot \delta z + u \frac{\partial \rho}{\partial x}\delta x \cdot \delta y \cdot \delta z$$

$$= \left(\rho \frac{\partial u}{\partial x} + u \frac{\partial \rho}{\partial x}\right)\delta x \cdot \delta y \cdot \delta z$$

$$= \frac{\partial}{\partial x}(\rho u)\delta x \cdot \delta y \cdot \delta z = \frac{\partial}{\partial x}(\rho u)\delta V$$



Similarly, the net mass rate leaving in the y and z directions are, $= \frac{\partial}{\partial y}(\rho v)\delta V$ and $= \frac{\partial}{\partial y}(\rho w)\delta V$ respectively.

The rate of accumulation of mass is given by

$$\frac{\partial}{\partial t}(\rho \delta x \cdot \delta y \cdot \delta z) = \frac{\partial \rho}{\partial t}\delta V$$

By mass balance,

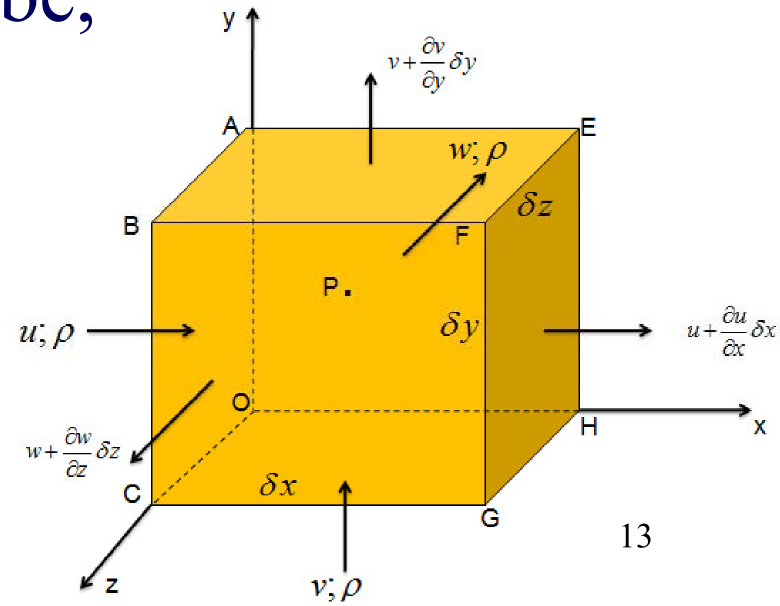
$$\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t}\right] \cdot \delta V = 0$$

Since δV≠0, However small the CV be,

$$= \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$

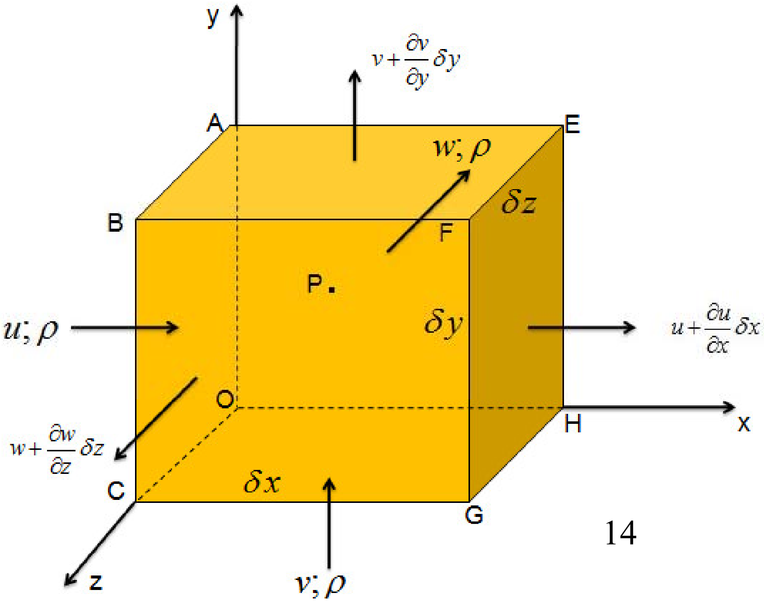
In Vector Form,

$$\nabla \cdot (\rho U) + \frac{\partial \rho}{\partial t} = 0$$



This is the continuity equation for

a) In 3D for a general fluid flows,
b) For compressible and incompressible fluids,
c) For viscous and inviscid flow,
d) For steady and unsteady flow,
e) For uniform and non-uniform flow since u,v,w are the local values for the continuity at a point.



For incompressible 3D flow, ρ = constant,

Whether steady or unsteady $\frac{\partial \rho}{\partial x} = 0 = \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial z} = \frac{\partial \rho}{\partial t}$

The continuity equation, then reduces to $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

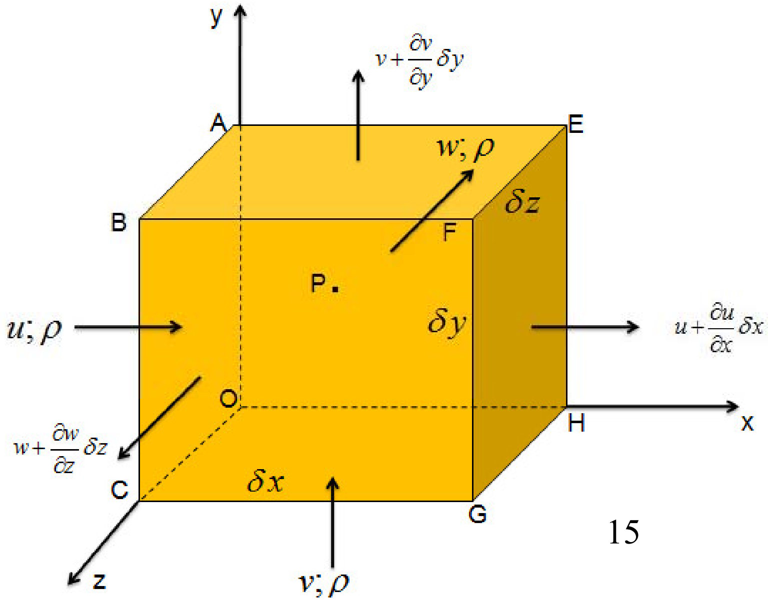
For incompressible 2D flow $\frac{\partial}{\partial z} \equiv 0$

For flow in x-y plane , continuity equation, reduces to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

For compressible but steady flow,

$$\frac{\partial}{\partial t} \equiv 0$$



The continuity equation, then reduces to

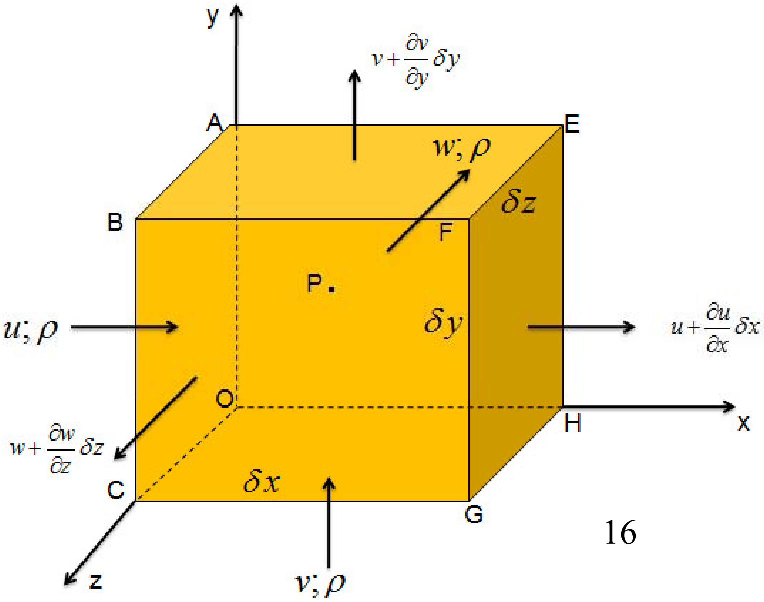
$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}=0$$

In addition

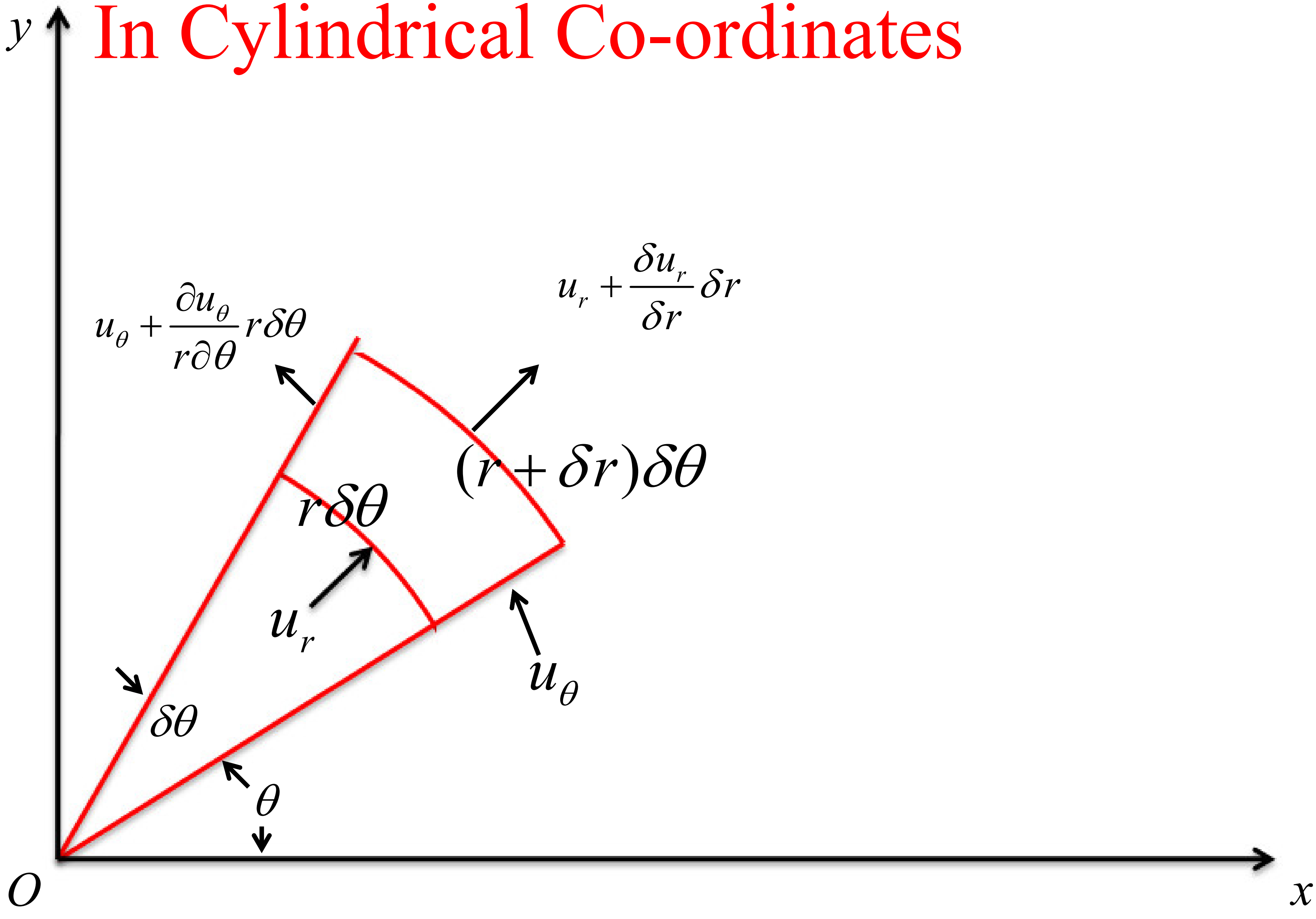
$$\frac{\partial}{\partial z}\equiv 0$$

For flow in x-y plane and the continuity equation reduces to

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0$$



Continuity Equation – Differential Form In Cylindrical Co-ordinates



Continuity Equation – Differential Form

- Mass flow rate entering radially at radius r $= u_r \rho \bullet r\delta\theta \bullet \delta z$
- Mass flow rate leaving radially at radius (r+dr)

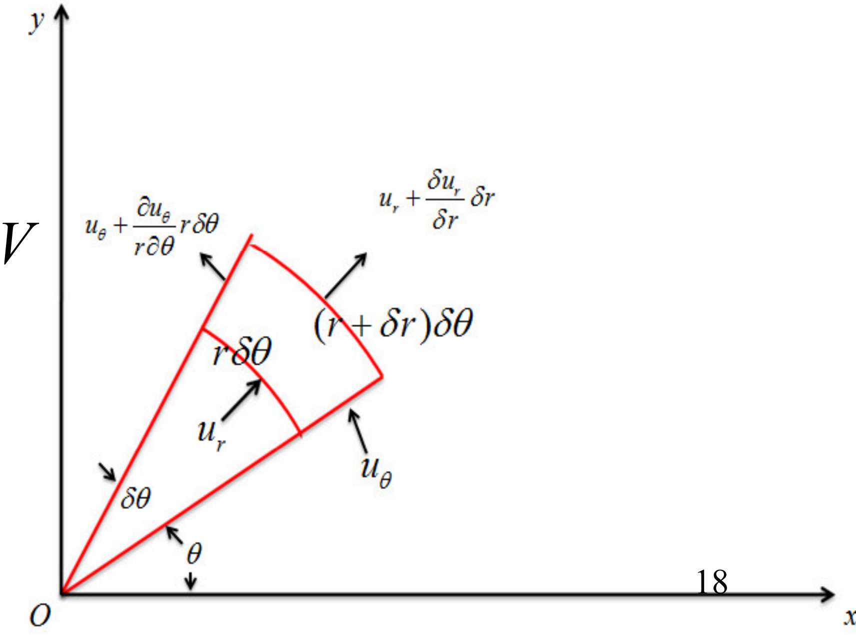
$$= \left(u_r + \frac{\partial u_r}{\partial r}\delta r \right) \rho \bullet \left(r + \delta r \right) \delta\theta \bullet \delta z$$

$$= u_r \rho \bullet r\delta\theta \bullet \delta z + u_r \rho \bullet \delta r \bullet \delta\theta \bullet \delta z + \frac{\partial u_r}{\partial r}\delta r \bullet \rho \bullet r \bullet \delta\theta \bullet \delta z + \text{ smaller terms}$$

- Hence, net mass rate leaving in the radial direction

$$= u_r \rho \bullet \delta r \bullet \delta\theta \bullet \delta z + \frac{\partial u_r}{\partial r}\delta r \bullet \rho \bullet r \bullet \delta\theta \bullet \delta z$$

$$= \left(\frac{u_r}{r} + \frac{\partial u_r}{\partial r} \right) \delta r \bullet \rho \bullet r \bullet \delta\theta \bullet \delta z = \left(\frac{u_r}{r} + \frac{\partial u_r}{\partial r} \right) \rho \bullet \delta V$$

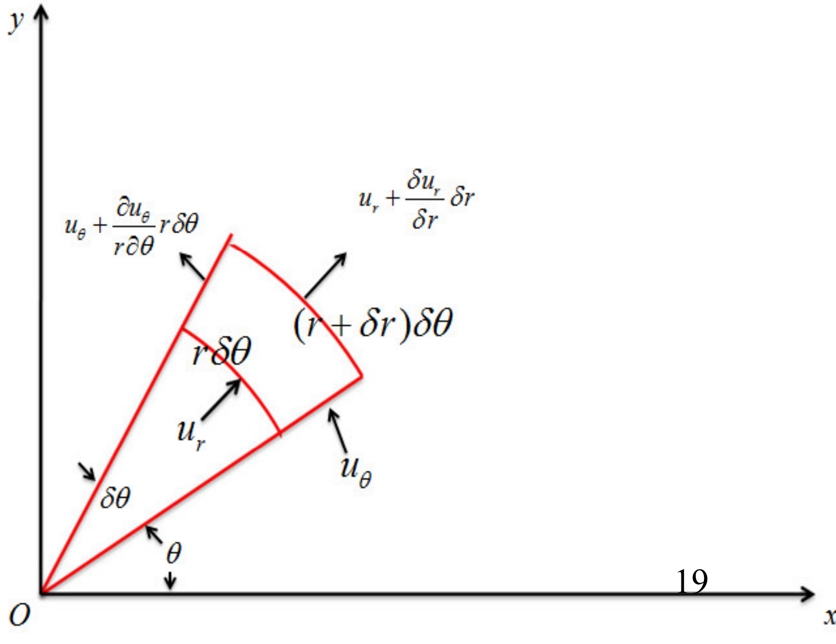


Continuity Equation – Differential Form

- Similarly, net mass flow rate leaving in the θ and z direction

$$=\left(u_\theta+\frac{\partial u_\theta}{r\partial\theta}\bullet r\delta\theta\right)\rho\bullet\delta r\bullet\delta z-u_\theta\rho\bullet\delta r\bullet\delta z=\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\rho\bullet\delta V$$

$$=\left(u_z+\frac{\partial u_z}{\partial z}\bullet\delta z\right)\rho\bullet\delta r\bullet r\delta\theta-u_z\rho\bullet\delta r\bullet r\delta\theta=\frac{\partial u_z}{\partial z}\rho\bullet\delta V$$



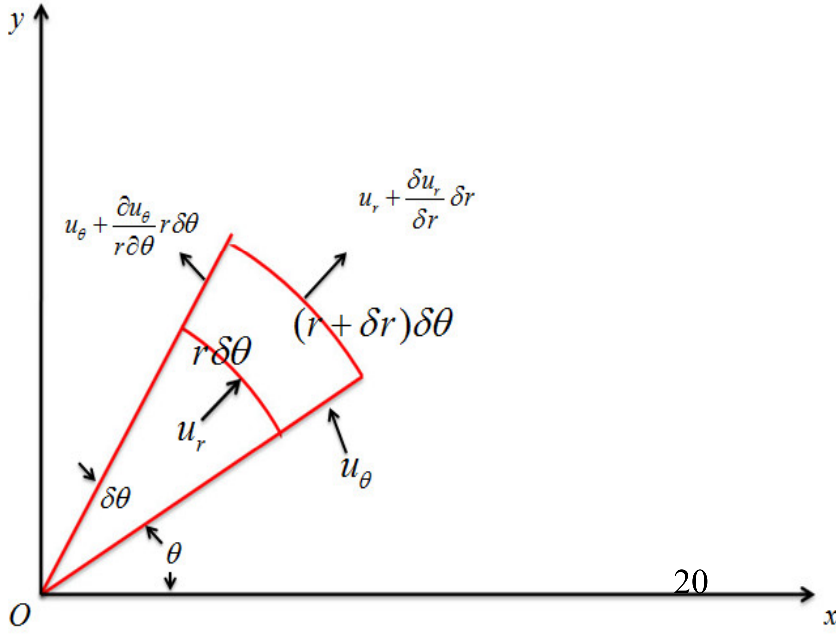
Continuity Equation – Differential Form

- There being no rate of accumulation of fluid in an incompressible flow, the net efflux across the control volume should be zero.

$$\left(\frac{u_r}{r}+\frac{\partial u_r}{\partial r}\right)\rho\bullet\delta V+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}\rho\bullet\delta V+\frac{\partial u_z}{\partial z}\rho\bullet\delta V=0$$

$$\frac{u_r}{r}+\frac{\partial u_r}{\partial r}+\frac{1}{r}\frac{\partial u_\theta}{\partial\theta}+\frac{\partial u_z}{\partial z}=0$$

$$\frac{\partial(ru_r)}{\partial r}+\frac{\partial u_\theta}{\partial\theta}+r\frac{\partial u_z}{\partial z}=0$$



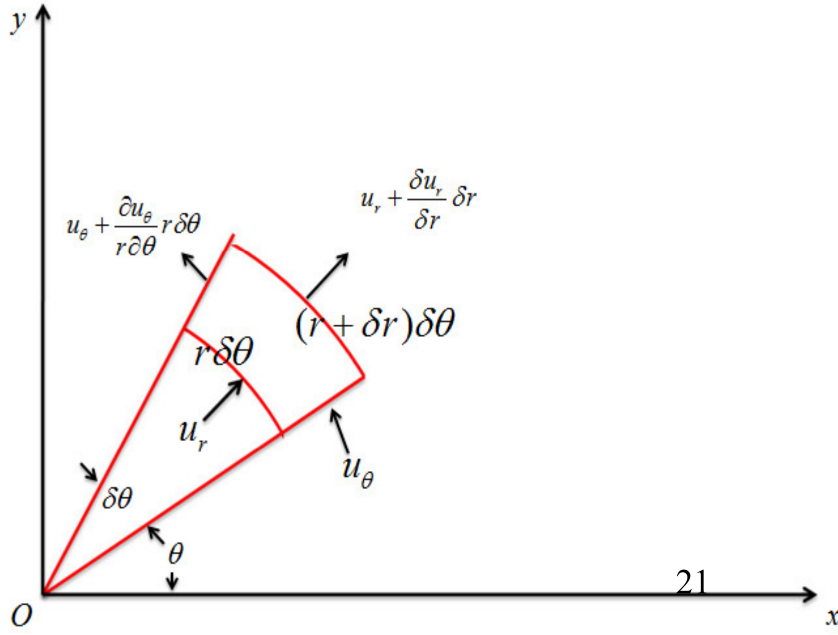
Continuity Equation – Differential Form

- If fluid undergoes 2 D flow:
  - i. $\frac{\partial}{\partial z} \equiv 0$ for flow in the r-θ plane, continuity equation becomes

$$\frac{\partial (r u_r)}{\partial r} + \frac{\partial u_\theta}{\partial \theta} = 0$$

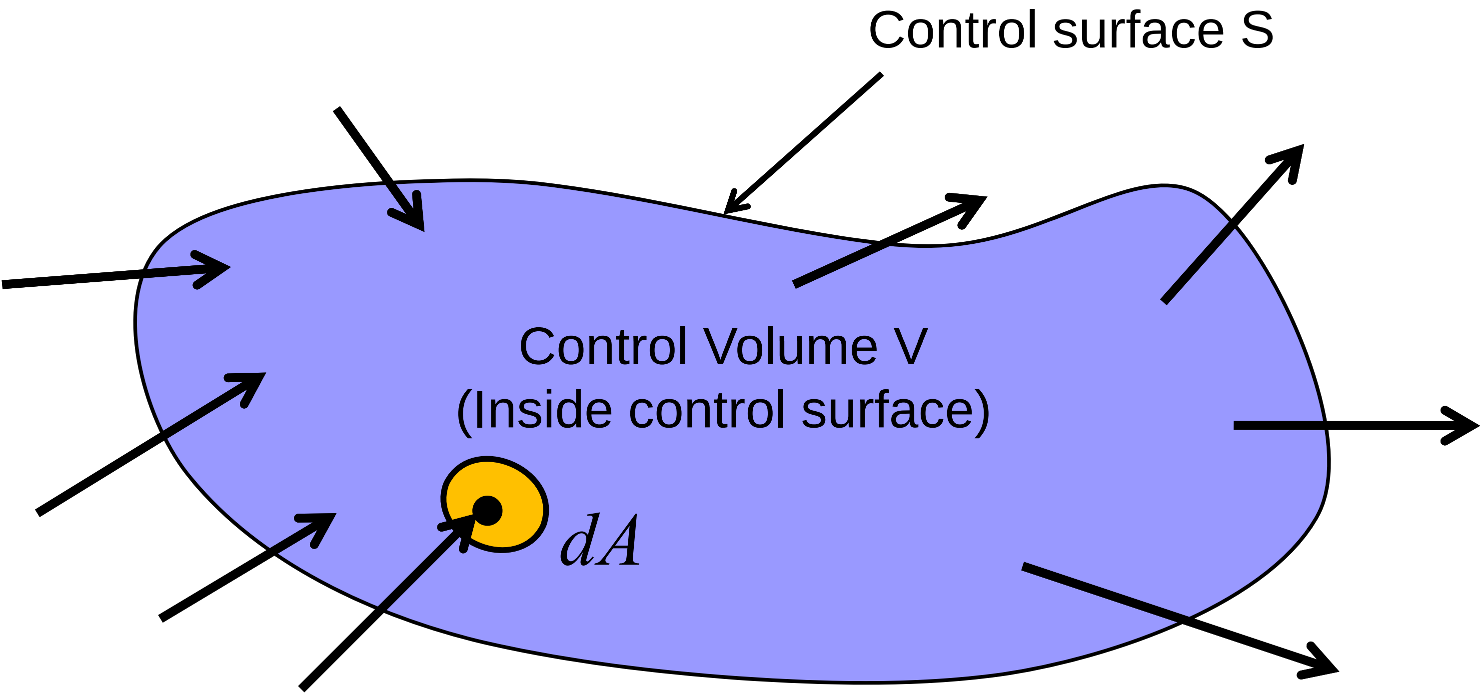
  - ii. $\frac{\partial}{\partial \theta} \equiv 0$ for axisymmetric flow, continuity equation becomes

$$\frac{1}{r}\frac{\partial (r u_r)}{\partial r} + \frac{\partial u_z}{\partial z} = 0$$



Continuity Equation – Integral Form

Integral form is applicable to a finite CV, virtually of any size and shape. If it has a volume V and is bounded by control surface S, net efflux of mass across control surface S and rate of accumulation of mass within control volume V must add to zero.



Continuity Equation – Integral Form

Integral form is applicable to a finite CV, virtually of any size and shape. If it has a volume V and is bounded by control surface S, net efflux of mass across control surface S and rate of accumulation of mass within control volume V must add to zero.

If the fluid crosses the elementary area dA with a velocity U then the net efflux across the control surface must be

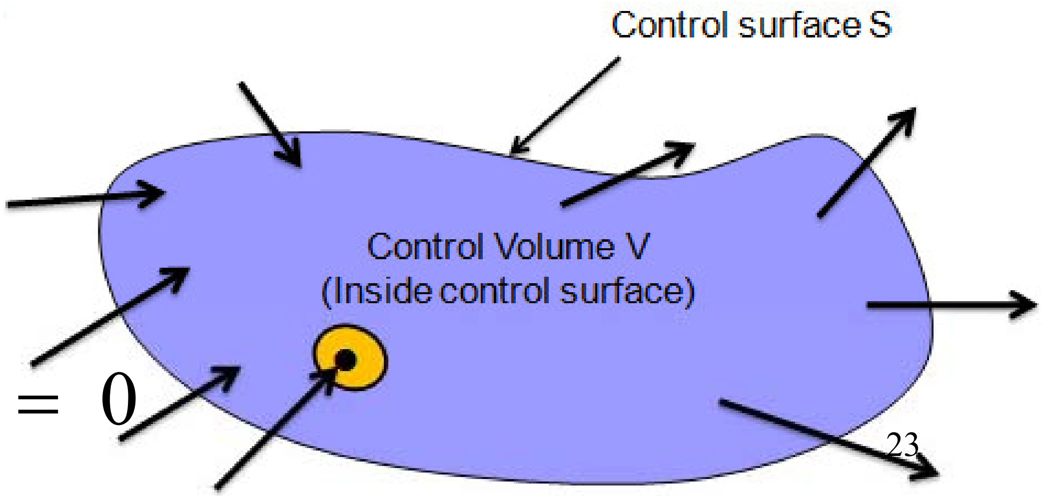
$$\int_s \rho U \cdot dA$$

The rate of mass accumulation within CV must be

$$\frac{\partial}{\partial t}\int \rho U \cdot dA$$

The continuity equation, then, becomes

$$\int_s \rho U \cdot dA + \frac{\partial}{\partial t}\int \rho U \cdot dA = 0$$



Continuity Equation – Integral Form

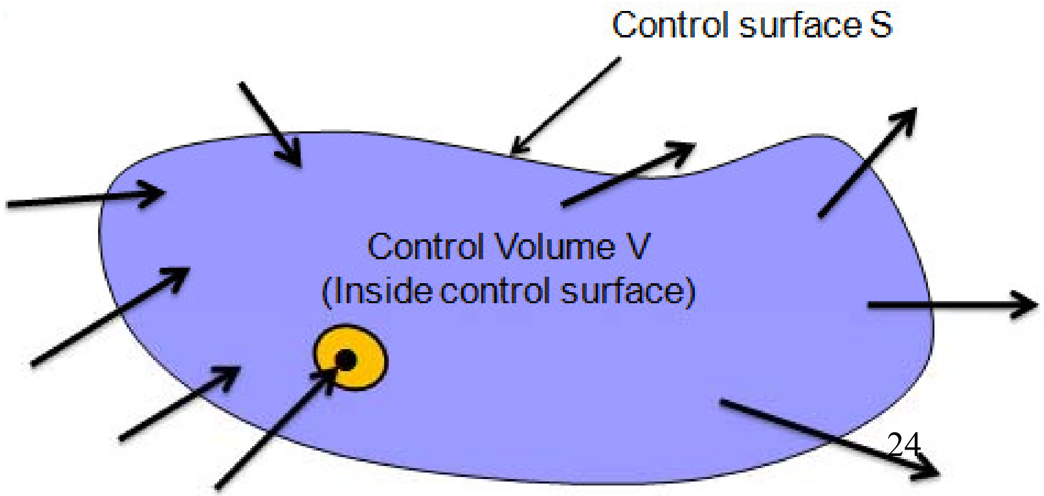
The integral form of the continuity equation can be shown to be equivalent to the differential form as follows,

The first integral of equation $\rho_1 A_1 U_1 = \rho_2 A_2 U_2$ can be converted into a volume integral by the use of divergence theorem. Since the CV does not change with time, the order of differentiation and integration in the second term can be interchanged.

The operation result in

$$\int \nabla\cdot(\rho U)dV + \int \frac{\partial \rho}{\partial t}dV = 0$$

$$\int \left\{\nabla\cdot(\rho U) + \frac{\partial \rho}{\partial t}\right\}dV = 0$$

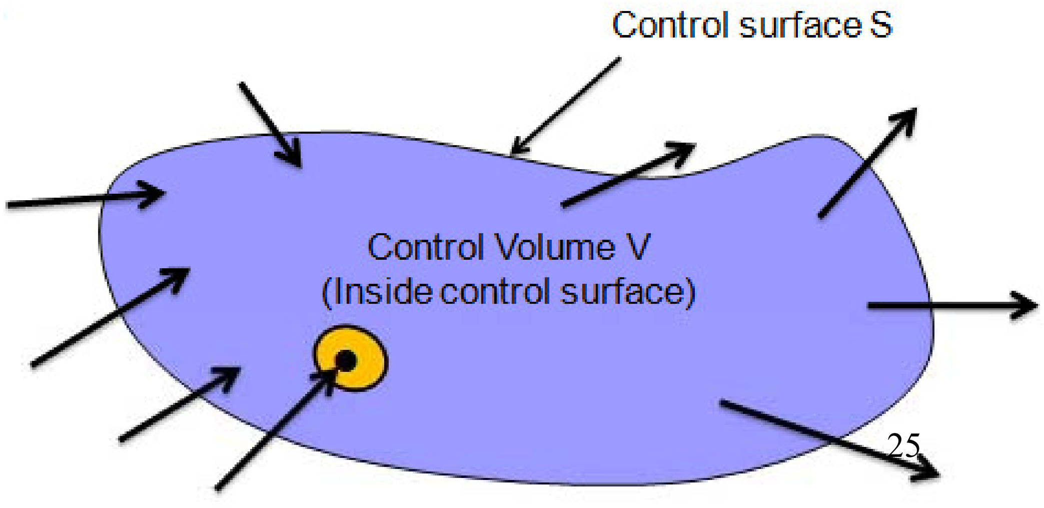


Continuity Equation – Integral Form

since the integral is zero for any arbitrary control volume, the integrand must vanish.

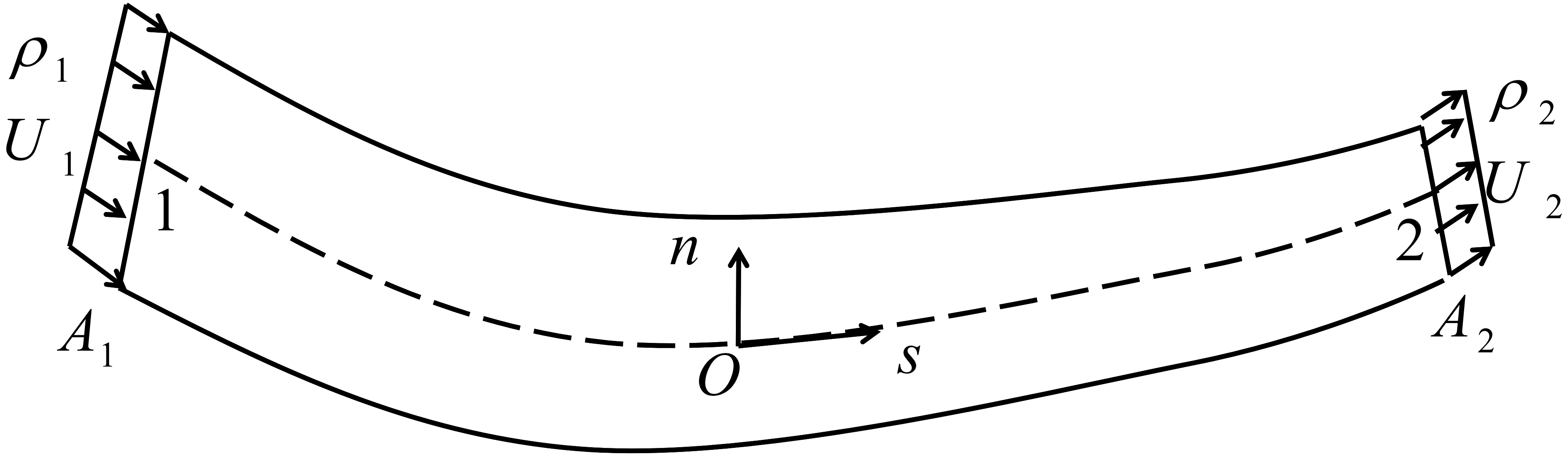
$$\nabla \cdot (\rho U) + \frac{\partial \rho}{\partial t} = 0$$

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$$



Continuity Equation – Integral Form

Now, consider the mass balance for flow through a control volume.



The continuity equation for steady incompressible flow reduces to

$$U\frac{\partial \rho}{\partial S}+\rho\frac{\partial U}{\partial S}=0$$

Continuity Equation – Integral Form

The fluid enters the c/s A_1 of CV with a uniform velocity U_1 and leaves the c/s A_2 of CV with a uniform velocity U_2. If flow is steady, there is no rate of accumulation within the CV.

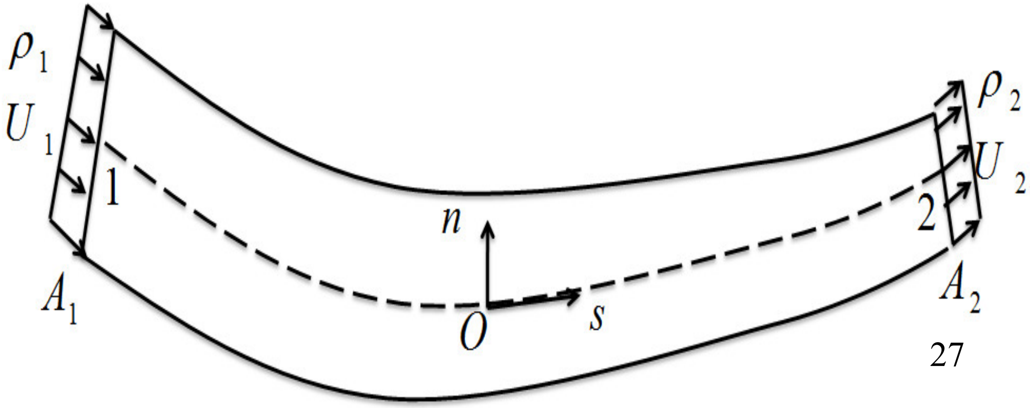
By Continuity, net efflux must be zero;

$$\rho_2 A_2 V_2 - \rho_1 A_1 V_1 = 0$$

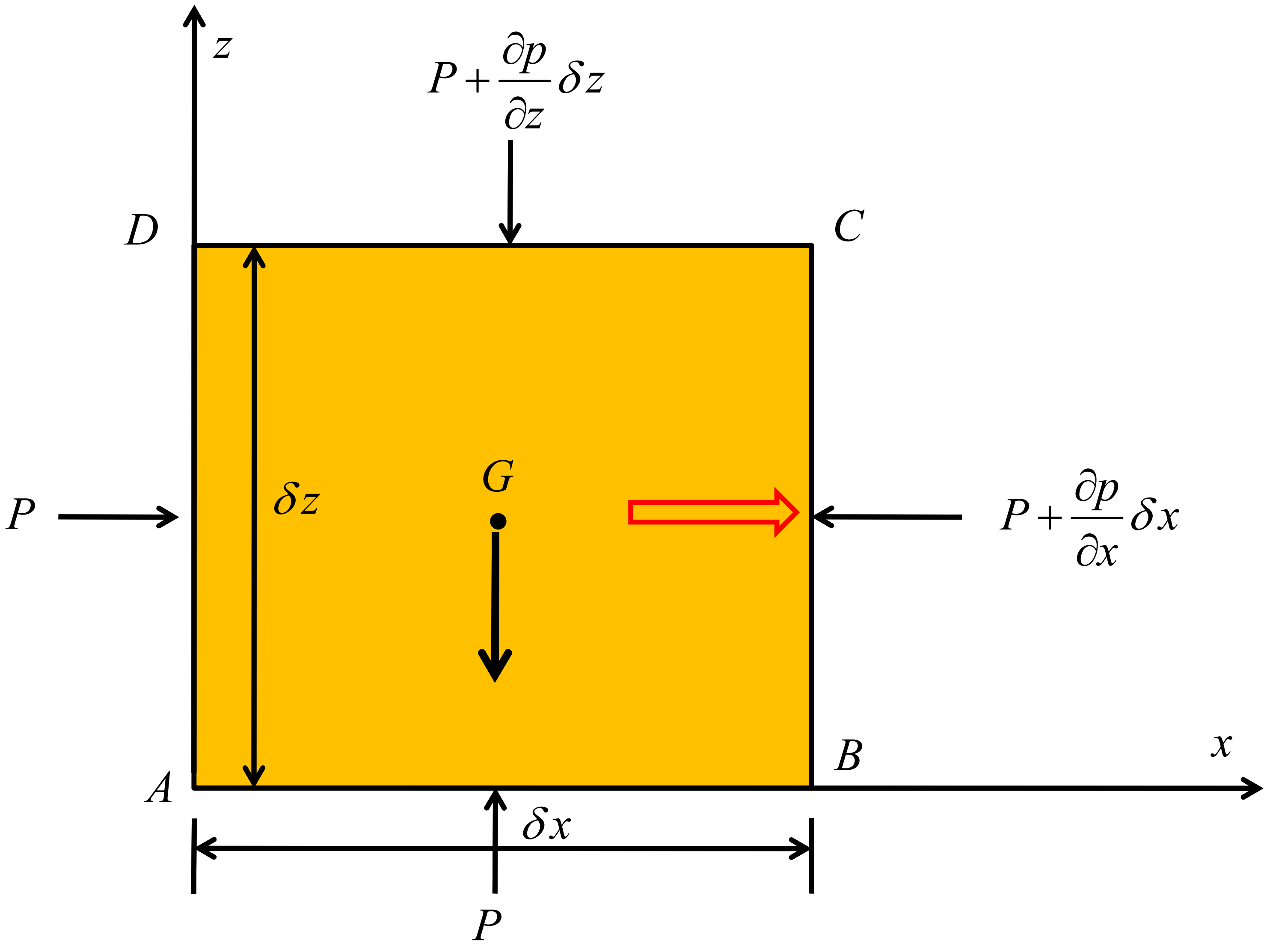
$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

If the flow is incompressible, $\rho_1 = \rho_2$, then

$$A_1 U_1 = A_2 U_2$$



Euler's Equation of Motion



Euler’s Equation of Motion

- Net surface force in x-direction,

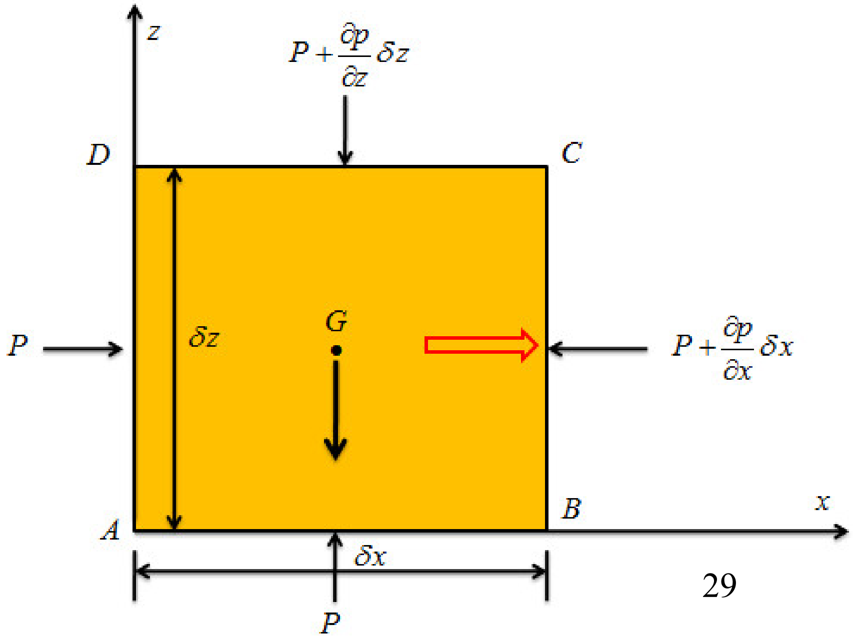
$$p\delta z\delta y-\left(p+\frac{\partial p}{\partial x}\delta x\right)\delta z\delta y=-\frac{\partial p}{\partial x}\delta x\delta z\delta y=-\frac{\partial p}{\partial x}\delta v$$

- Net surface force plus body force in *z*- direction,

$$=-\frac{\partial u}{\partial z}\delta V-\rho g\delta V$$

- Recalling the acceleration components for steady flow in *x*- direction

$$a_x=u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}$$



Euler's Equation of Motion

- In z – direction, $a_z = u\dfrac{\partial w}{\partial x} + w\dfrac{\partial w}{\partial z}$
- The Newton's second law in differential form

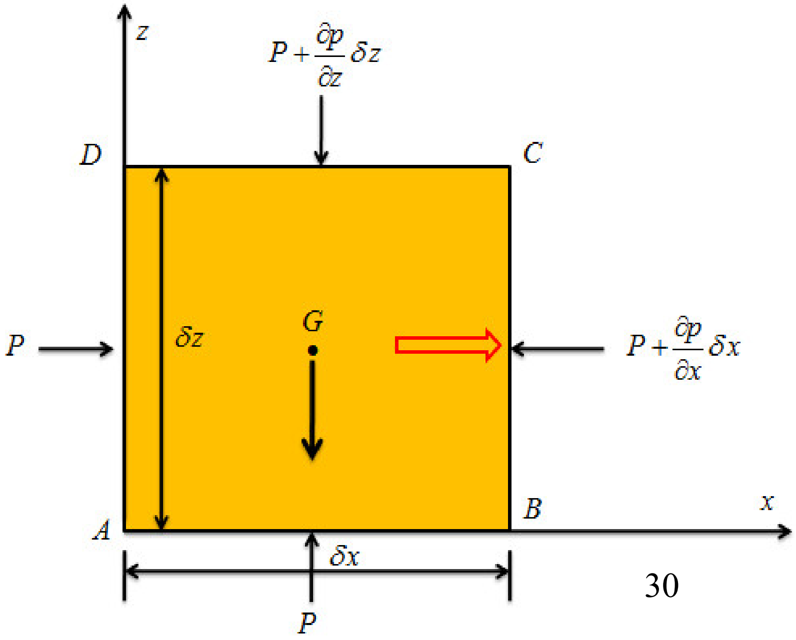
$$dF = dm\frac{DU}{Dt} = mass \times total.acceleration$$

- Along *x*-axis,

$$\left(u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z}\right)\rho\delta V = -\frac{\partial p}{\partial x}\delta V$$

- Along *z*-axis,

$$\left(u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z}\right)\rho\delta V = -\frac{\partial p}{\partial z}\delta V - \rho g\delta V$$



Euler's Equation of Motion

- For 2D steady flow of an inviscid fluid in a vertical plane, Euler's momentum equations are

$$u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g$$

Euler's Equation of Motion

- If fluid flow occurs under the action of body forces then B_x and B_z are external force components per unit mass

$$u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}=B_x-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}=B_z-\frac{1}{\rho}\frac{\partial p}{\partial z}$$

Euler’s Equation of Motion

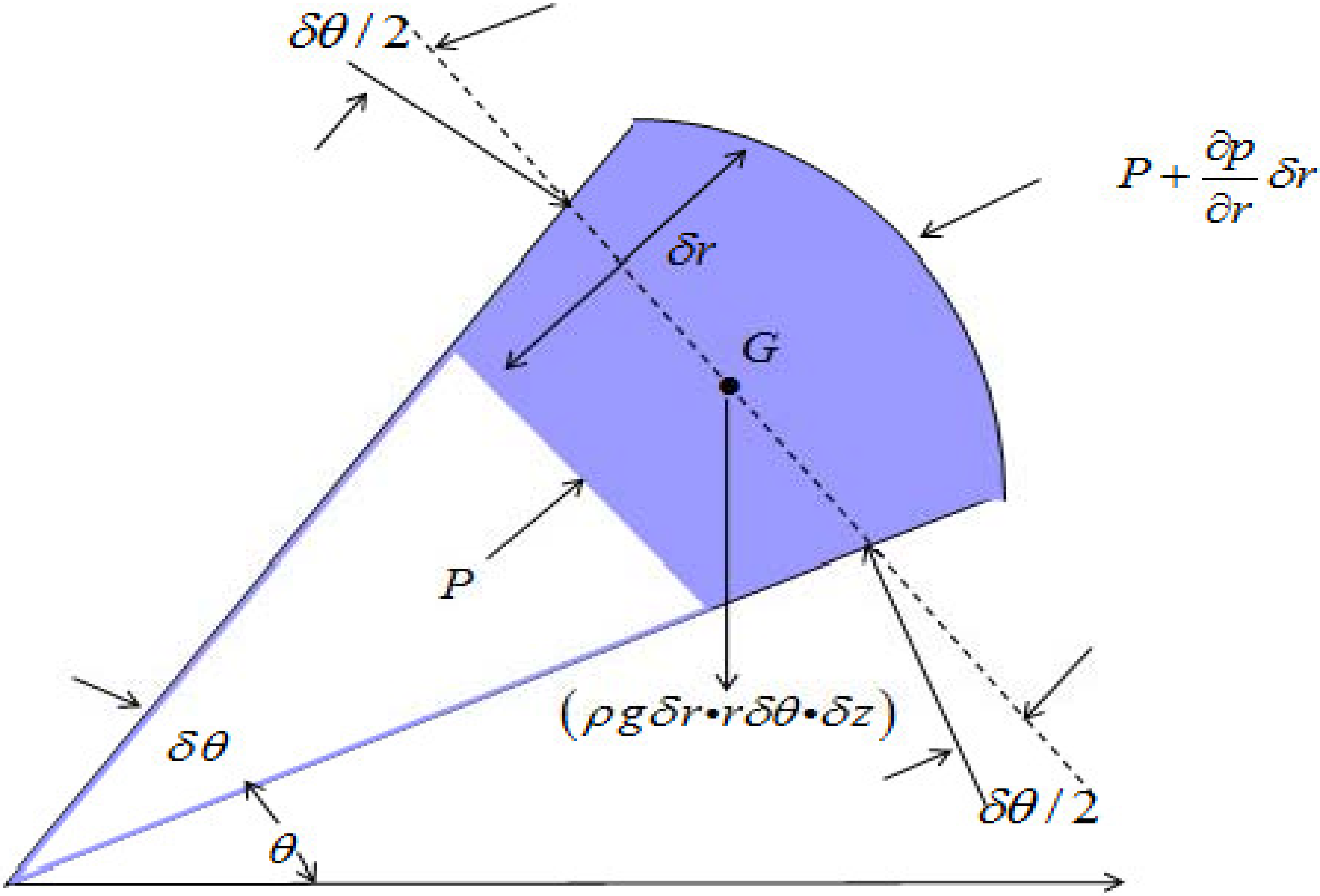
- Euler’s momentum equation for 3D inviscid steady flow -

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=B_x-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=B_y-\frac{1}{\rho}\frac{\partial p}{\partial y}$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=B_z-\frac{1}{\rho}\frac{\partial p}{\partial z}$$

Euler’s Equation of Motion



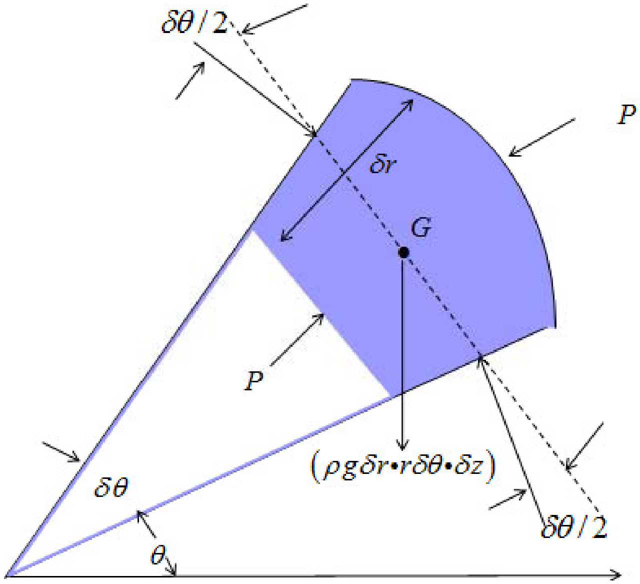
Euler's Equation of Motion

- Euler's momentum equation for cylindrical co-ordinates for an axisymmetric steady flow –

$$u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_z}{\partial z} + \frac{u_\theta{}^2}{r} = B_r - \frac{1}{\rho}\frac{\partial p}{\partial r}$$

$$u_r \frac{\partial u_\theta}{\partial r} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} = B_\theta$$

$$u_r \frac{\partial u_z}{\partial z} + u_z \frac{\partial u_z}{\partial r} = B_z - \frac{1}{\rho}\frac{\partial p}{\partial z}$$



- Where, B_r, B_θ and B_z are body forces per unit mass in r, θ and z direction respectively.

Bernoulli Equation: Derivation from the Euler's Equation

- Bernoulli's equation can be written as

$$\int \frac{dP}{\rho} + \frac{V^2}{2} + gz = \text{constant}$$

- The very fact that it can be derived from Euler's equation means that it is valid for inviscid fluid flow and it is contained in Euler's equation!
- Two different cases are considered in process of derivation which results in familiar form of Bernoulli's equation.

Bernoulli Equation

- Case (a) Along a Streamline in an Inviscid and Steady Flow.
- Case (b) Between Any Two Points in a Potential/ Irrotational and Steady Flow.
- For a 2D steady flow in presence of gravitational forces, Euler's equations are

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}.......................(i)$$

$$-g-\frac{1}{\rho}\frac{\partial p}{\partial z}=u\frac{\partial w}{\partial x}+w\frac{\partial w}{\partial z}.....................(ii)$$

Bernoulli Equation

- Multiplying (i) by δx and (ii) by δz and adding them,

$$-\frac{1}{\rho}\left(\frac{\partial p}{\partial x}\delta x+\frac{\partial p}{\partial z}\delta z\right)-g\delta z=u\frac{\partial u}{\partial x}\delta x+w\frac{\partial u}{\partial z}\delta x+u\frac{\partial w}{\partial x}\delta z+w\frac{\partial w}{\partial z}\delta z.....(iii)$$

- Left hand side term is written as

$$-\frac{dp}{\rho}-gdz$$

Bernoulli Equation

- For case (a) Along a Streamline in an Inviscid and Steady Flow, invoke the equation of a streamline in x-z plane. $u\delta z - w\delta x = 0$

  or $u\delta z = w\delta x$

- RHS of (iii) becomes

$$\left(u\frac{\partial u}{\partial x}\delta x + u\frac{\partial u}{\partial z}\delta z \right) + \left(w\frac{\partial w}{\partial x}\delta x + w\frac{\partial w}{\partial z}\delta z \right)$$

$$= udu + wdw = d\left(\frac{u^2 + w^2}{2} \right) = d\left(\frac{U^2}{2} \right)$$

Bernoulli Equation

- Case (b) Between Any Two Points in a Potential/ Irrotational and Steady Flow, invoke the condition of irrotationality in x-z plane

$$\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0$$

$$\frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}$$

Bernoulli Equation

- RHS of (iii) becomes

$$u\frac{\partial u}{\partial x}\delta x + w\frac{\partial w}{\partial x}\delta x + u\frac{\partial u}{\partial z}\delta z + w\frac{\partial w}{\partial z}\delta z$$

$$= \left(u\frac{\partial u}{\partial x}\delta x + u\frac{\partial u}{\partial z}\delta z\right) + \left(w\frac{\partial w}{\partial x}\delta x + w\frac{\partial w}{\partial z}\delta z\right)$$

$$= udu + wdw = d\left(\frac{u^2 + w^2}{2}\right) = d\left(\frac{U^2}{2}\right)$$

Which is precisely same as case (a)

Bernoulli Equation

- For either (a) or (b), it can be written as

$$-\frac{dp}{\rho} - g dz = d\left(\frac{U^2}{2}\right)$$

- Euler's equation for the steady flow of an inviscid fluid may be written, in vector notation, as

$$-\frac{\nabla p}{\rho} - g\nabla z = U\frac{\partial U}{\partial s}$$

Bernoulli Equation

- If *s* is coordinate along a streamline or it is along any line in an irrotational flow, take dot product of each term with an elementary displacement *ds* along *s*. Thus,

$$-\frac{\nabla p}{\rho}\bullet ds - g\nabla z\bullet ds = U\frac{\partial U}{\partial s}\bullet ds$$

$$\nabla p\bullet ds = dp$$

$$\nabla z\bullet ds = dz$$

- RHS terms simplifies to UdU or d(U^2/2)

Bernoulli Equation

- Consequently it results in same direction:

$$-\frac{dp}{\rho}-gdz=d\left(\frac{U^2}{2}\right)$$

- On rearrangement and integration,

$$\int\frac{dp}{\rho}+d\left(\frac{U^2}{2}\right)+gdz=\text{constant}$$

Which is precisely same as case (a)

Bernoulli Equation

- For different cases

1. *For an incompressible fluid flow, ρ =constant*

$$\frac{p}{\rho g}+\frac{U^2}{2g}+z=\text{constant}$$

2. *For a compressible fluid flow, undergoing an adiabatic process*

$$p=C\rho^{\gamma}$$

$$dp=\gamma C\rho^{\gamma-1}d\rho$$

$$\frac{\int dp}{\rho g}=\frac{\gamma}{g}C\int\rho^{\gamma-2}d\rho=\frac{\gamma}{g}\frac{p}{\rho^{\gamma}}\frac{\rho^{\gamma-1}}{(\gamma-1)}=\frac{\gamma}{(\gamma-1)}\frac{p}{g\rho}$$

Bernoulli Equation

- The Bernoulli's equation appears as

$$\frac{\gamma}{\gamma-1}\frac{p}{\rho}+\frac{U^2}{2}+gz=\text{constant}$$

- Following are different assumptions made in derivation of above equation –
  1) inviscid flow, $\mu=0$
  2) steady flow, $\frac{\partial}{\partial t}=0$
  3) either the flow is along a streamline or the flow is irrotational
  4) 2D flow, in presence of gravitational forces.

Navier-Stokes Equation of Motion

- It is the general momentum equation for compressible or incompressible, viscous or inviscid flows.
- When shear stress on faces of a control volume are included in the force-balance together with the normal stresses, Newton's second law of motion provides

$$\rho\frac{Du}{Dt}=\rho B_x+\frac{\partial\tau_{xx}}{\partial x}+\frac{\partial\tau_{yx}}{\partial y}+\frac{\partial\tau_{zx}}{\partial z}$$

$$\rho\frac{Dv}{Dt}=\rho B_y+\frac{\partial\tau_{yy}}{\partial y}+\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{yz}}{\partial z}$$

$$\rho\frac{Dw}{Dt}=\rho B_z+\frac{\partial\tau_{zz}}{\partial z}+\frac{\partial\tau_{zx}}{\partial x}+\frac{\partial\tau_{zy}}{\partial y}$$

Navier-Stokes Equation of Motion

- By Stoke’s law of velocity for an incompressible fluid,

$$\sigma_{xx} = -p + 2\mu \frac{\partial u}{\partial x}$$

$$\sigma_{yy} = -p + 2\mu \frac{\partial v}{\partial y}$$

$$\sigma_{zz} = -p + 2\mu \frac{\partial w}{\partial z}$$

$$\tau_{xy} = \mu \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right)$$

$$\tau_{zy} = \mu \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z} \right)$$

$$\tau_{zx} = \mu \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} \right)$$

Navier-Stokes Equation of Motion

- From first three relations,

$$p = -\frac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}$$

- By continuity,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

Navier-Stokes Equation of Motion

- On substitution ,

$$\rho\frac{Du}{Dt} = \rho B_x - \frac{\partial p}{\partial x} + \mu\nabla^2 u$$

$$\rho\frac{Dv}{Dt} = \rho B_y - \frac{\partial p}{\partial y} + \mu\nabla^2 v$$

$$\rho\frac{Dw}{Dt} = \rho B_z - \frac{\partial p}{\partial z} + \mu\nabla^2 w$$

  - This is Navier-Stokes equation for a 3D incompressible fluid flow.